∴ $y^2-3x^2=-\frac{1}{3}(4D^2)$. This is the equation of an hyperbola referred to its center o' as the origin. To write it in the ordinary form, that is, in terms of the transverse and conjugate axes, multiply each term by C; *i. e.* let $\sqrt{C}$=semi-transverse axis. Thus $Cy^2-3Cx^2=-\frac{1}{3}(4CD^2)$.

When in this form the product of the coefficients of the x^2 and y^2 terms should be equal to the remaining term. That is $-3C^2=-\frac{1}{3}(4CD^2)$......(III). ∴ $C=\frac{1}{9}(4D^2)$, and equation III becomes $\frac{1}{9}(4D^2)y^2-\frac{1}{3}(4D^2)x^2=-\frac{1}{27}(16D^4)$.

The semi-transverse axis $=\sqrt{[\frac{1}{9}(4D^2)]}=\frac{1}{3}(2D)$. The semi-conjugate axis $=\sqrt{[\frac{1}{3}(4D^2)]}=2D/\sqrt{3}$.

Since the distance from the center of the curve to either focus is equal to the square root of the sum of the squares of the semi-axes, the distance from o' to either focus $=\sqrt{\{[\frac{1}{9}(4D^2)]+[\frac{1}{3}(5D^2)]\}}=\frac{1}{3}(4D)$. We can therefore make the following construction—Fig. 2. Draw ad the chord of the arc acd. Trisect ad at o' and k. Produce da to l, making $al=ao'=o'k=kd$. With ak as a transverse axis, and l and d as foci, construct the branch of the hyperbola $kcc'c''$, which will intersect all arcs having the common chord ad at c, c', c'', etc., making the arcs cd, $c'd$, $c''d$, respectively, equal to one-third of the arcs acd, $ac'd$, $ac''d$, etc.

CALCULUS.

67. Proposed by BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, O.

A man starts to walk at a uniform rate across a draw-bridge just as it begins to move. He walks the full length of the bridge and back, in the same time that it takes the bridge to make a half revolution. How for does he ride, the length of the bridge being 250 feet, and its velocity uniform about a center axis?

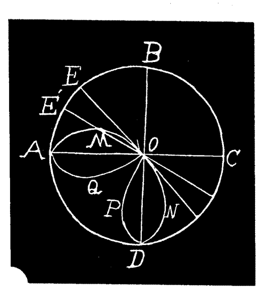
I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

In my solution of problem 45, I deduced the equation, $\rho=r(m-\theta)/m$. Let $m=\frac{1}{4}\pi$. ∴ $\rho=(r/\pi)(\pi-4\theta)$ is the equation to the man's path in space.

$$\therefore S=\frac{2r}{\pi}\int_0^{\frac{1}{2}\pi}\sqrt{16+(\pi-4\theta)^2}\,d\theta=\tfrac{1}{2}r\sqrt{16+\pi^2}+\frac{8r}{\pi}\left(\frac{\pi+\sqrt{(16+\pi^2)}}{4}\right)$$

$$=\tfrac{125}{2}\sqrt{16+\pi^2}+\frac{1000}{\pi}\log\frac{\pi+\sqrt{(16+\pi^2)}}{4}. \quad \therefore S=547.468 \text{ feet.}$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.



According to the conditions of the problem, A being the starting point of the man, he will be at O, going in the curve AMO, when the draw-bridge has turned through $\angle AOE=45°$; when the latter has turned through $\angle AOB=90°$, the man is in D, having passed through the curve OND; after the bridge has turned through $\angle AOF=135°$, the man is at O again, having moved through the curve DPO, and after the

draw-bridge has made a semi-revolution, he is back at A, having been swept through the curve AQO. Choosing O for the origin of polar coödinates, we have for the curve to the polar equation $r=4R\theta/\pi$, R being the radius of the revolving draw-bridge. The required path of the man is

$$=4\int_0^{\frac{1}{4}\pi}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta=\frac{16R}{\pi}\int_0^{\frac{1}{4}\pi}\sqrt{\theta^2+1}\,d\theta=\frac{8R}{\pi}\Big[\theta\sqrt{\theta^2+1}+$$

$$\log(\theta+\sqrt{\theta^2+1})\Big]_0^{\frac{1}{4}\pi}=\tfrac{1}{2}R\left[\sqrt{\pi^2+16}+\frac{16}{\pi}\log\left(\frac{\pi+\sqrt{(\pi^2+16)}}{4}\right)\right];$$

for $R=125$ feet, we get this length$=547.45$ feet.

III. **Solution by J. M. BANDY, A. M., Civil Engineer for the Roads and Bridges of Guilford County, Greensboro, N. C.**

Let M be any position of the man, let $(\rho,\ \theta)$ denote the polar coördinates of M, and let r=radius of the bridge. When A shall have revolved through $45°$ the man will be at O, the center; and the limits of θ for $\frac{1}{4}$ the curve are 0 and $\frac{1}{4}\pi$.

Since the man and the motion of the bridge are uniform, AE' and $E'M$ are in a constant ratio, which denote by n. But $AE'=r\theta$, $E'M=r-\rho$, and $n=\frac{1}{4}\pi$. Hence, $r[1-(4/\pi)\theta]$......(1). By the theory of curves,

$$S=\int\left(\rho^2+\frac{d\rho^2}{d\theta^2}\right)^{\frac{1}{2}}d\theta.\ \text{From (1), } \rho^2=r^2[1-(4/\pi)\theta]^2,\ \text{and } \frac{d\rho^2}{d\theta^2}=\frac{4r^2}{\pi^2}.$$

Substituting in formula,

$$S=4\int_0^{\frac{1}{4}\pi}\left[\frac{4r^2}{\pi^2}+\frac{r^2}{\pi^2}(\pi-4\theta)^2\right]^{\frac{1}{2}}d\theta,=\frac{4r}{\pi}\int_0^{\frac{1}{4}\pi}[4+(\pi-4\theta)^2]^{\frac{1}{2}}\,d\theta,$$

$$=\frac{4r}{\pi}\left[\frac{(\pi-4\theta)}{2}\sqrt{4+(\pi-4\theta)^2}+\tfrac{1}{2}\log\left((\pi-4\theta)+\sqrt{4+(\pi-4\theta)^2}\right)\right]_0^{\frac{1}{4}\pi},$$

$$=\tfrac{1}{2}r\sqrt{16+\pi^2}+(4r/\pi)\log\left(\frac{\pi+\sqrt{(16+\pi^2)}}{4}\right)$$

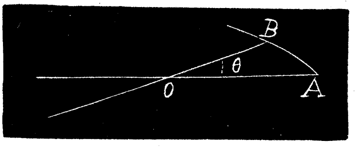
$$=\tfrac{125}{2}\sqrt{16+\pi^2}+\frac{1000}{\pi}\log\left(\frac{\pi+\sqrt{(16+\pi^2)}}{4}\right).$$

$S=547.45$ feet.

IV. **Solution by C. W. M. BLACK, A. M., Professor of Mathematics, Wesleyan Academy, Wilbraham, Mass.**

Let a=number of complete revolutions made while he walks the full length. $b=2OA$=length of draw-bridge. Let AB be curve traversed. $OB=\rho$, $\angle AOB=\theta$.

Then $\frac{1}{2}b-\rho : b=\theta : 2a\pi$.



$$\rho=b\left(\tfrac{1}{2}-\frac{\theta}{2a\pi}\right).\quad \frac{d\rho}{d\theta}=-\frac{b}{2a\pi}.$$

s (distrance traversed in passing to end of bridge)

$$=\int_0^{2a\pi}\left[b^2\left(\tfrac{1}{2}-\frac{\theta}{2a\pi}\right)^2+\frac{b^2}{4a^2\pi^2}\right]^{\frac{1}{2}}d\theta=(\tfrac{1}{2}b)\sqrt{1+a^2\pi^2}-$$

$$\frac{b}{2a\pi}\log_e(\sqrt{1+a^2\pi^2}-a\pi).\quad b=250.\quad a=\tfrac{1}{4}.$$

$$\therefore\ 2s=250[1+(\tfrac{1}{16}\pi^2)]-(1000/\pi)\log_e\{\sqrt{[1+(\tfrac{1}{16}\pi^2)]}-\tfrac{1}{4}\pi\}=547.6+\ \text{feet}.$$

[See also solutions of problems 41, 45, and 50, published in previous numbers of MONTHLY. EDITOR.]

MECHANICS.

56. Proposed by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics in Manual Training School, Philadelphia, Pa.

"Hey-diddle-diddle, the cat and the fiddle,
The cow jumped over the moon."

Taking the weight of the cow to be 600 pounds, the initial resistance of the air to be 100 pounds and varying as the square of the velocity, find the initial and final velocities, and the times of rising and falling.

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let R=radius of earth=20924640 feet,

$h=60.27R=1261128052.8$ feet=distance to moon,

$g=32.2$ feet, $W=600$ pounds, $\mu v^2=100$ pounds$=\frac{1}{6}W$,

t=time of ascent; t_1=time of descent, v=initial velocity, v_1=final velocity. Then $\mu=W/6v^2$; $1/k=\sqrt{(W/\mu)}=v\sqrt{6}$. $\therefore\ k=1/v\sqrt{6}$.

$h=(1/2gk^2)\log(1+v^2k^2)$, $t=(1/gk)\tan^{-1}vk$,

$t_1=(1/gk)\log[\sqrt{(1+v^2k^2)}+vk]$, $v_1=v/\sqrt{(1+v^2k^2)}$.

$\therefore\ (1+v^2k^2)=\frac{7}{6}$, $vk=1/\sqrt{6}$.

$\therefore\ v^2=gh/3\log(\frac{7}{6})$, $v=449657$ feet=85.16 miles per second.

$v_1=(\frac{6}{7})v=73$ miles per second.

$t=[v\sqrt{(6)}/g]\tan^{-1}(1/\sqrt{6})=13258.2$ seconds=3 hours, 40 minutes, 58.2 seconds.

$t_1=[v\sqrt{(6)}/g]\log\{[\sqrt{(7)}+1]/[\sqrt{6}]\}=13603.7$ seconds=3 hours, 46 minutes, 43.7 seconds.

In the above we have considered the resisting medium as extending to the moon.

57. Proposed by J. C. NAGLE, A. M., M. C. E., Professor of Civil Engineering, Agricultural and Mechanical College of Texas, College Station, Texas.

Over the intersection of two inclined planes slides a cord of uniform mass throughout its length. Find the equation of the path described by its center of gravity.

[No solution of this problem has been received. EDITOR.]